REPORT ON
ACTIVE LEARNING APPROACHES IN
MATHEMATICS INSTRUCTION:
PRACTICE & ASSESSMENT SYMPOSIUM
AUGUST 2–3, 2016
Recent research has established the positive impact of active learning teaching methods. A meta-study titled "Active Learning Increases student performance in science, engineering, and mathematics" (by Freeman, Eddy, McDonough and their colleagues), encouraged moving beyond comparisons between active learning vs. lecture format and towards exploring the best methods with an active learning classroom. The group wrote that their results "raise questions about the continued use of traditional lecturing as a control in research studies, and support active learning as the preferred, empirically validated teaching practice in regular classrooms." Their meta-analysis of 225 previous studies led them to conclude that "the data indicate that active learning increases student performance across the STEM disciplines" and that an increase in STEM students could be achieved by "abandoning traditional lecturing in favor of active learning."

This symposium addressed the critical national goal expressed by the President's Council of Advisors on Science and Technology (PCAST) in their call for a 33% increase in the number of science, technology, engineering, and mathematics (STEM) bachelor’s degrees. A principle tactic of the symposium was to "jump start" new implementation and evaluation research on active learning and IBL teaching methods in undergraduate mathematics. The research results discussed should encourage the creation of new active learning programs. Since active learning has been found to benefit students by improving communication and critical thinking skills, especially among traditionally underserved groups, the long-term impact of the research symposium should be to increase the number of students pursuing STEM majors and careers.

The symposium provided the logical step of moving new research towards exploring which active learning methods work best, especially for underrepresented groups of students, and explored ways to overcome current barriers that prevent active learning techniques from being attempted at existing mathematics programs.

This symposium brought together experts in education and economics research methods with active learning mathematics practitioners and departmental leaders to review current active learning evaluation efforts, clarify barriers to implementing new programs, and frame ways to increase national research and evaluation activities. The symposium attempted to identify key research issue areas and potential project teams to create new active learning initiatives. The symposium goal was to increase national mathematics active learning evaluation and associated research activities in order to strengthen mathematics teaching. The symposium consisted of four panel presentations by 12 presenters to a group of 81 invited and public attendees. Subsequently, key findings of the symposium have been circulated at national and regional mathematics conferences. A summary of symposium findings was compiled and posted on line for public use. In the following pages we summarize the content and thrust of each of the presentations along with relevant references for further information.

Ronald G. Douglas is a Distinguished Professor of Mathematics at Texas A&M University, a Guggenheim Fellow, Sloan Fellow, a fellow of the American Mathematical Society, and fellow of the American Association for the Advancement of Science. He has had 24 doctoral students and over 10 postdoctoral students, and published more than 150 research papers and four books. He led the calculus reform movement in the late 1980’s and an NRC study of doctoral education in the mathematical sciences in 1991. He has served as department head, dean, and provost.
### Active Approaches in Learning Mathematics Instruction: Practice and Assessment Symposium

The event was held in Columbus, Ohio August 2-3, 2016 and featured presentations from a dozen practitioners and was attended by over 80 participants. The symposium was funded from grants gifted by The National Science Foundation, the Educational Advancement Foundation, and the Sloan Foundation.

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The mathematical community has come to recognize the importance of the use of active learning in undergraduate mathematics classes. We see this recognition in the Common Vision report issued jointly by the AMATYC, AMS, ASA, MAA; and SIAM. In July, 2016, the presidents of these and ten additional mathematical societies represented by the Conference Board of the Mathematical Sciences signed onto a statement endorsing active learning in post-secondary mathematics education. It includes the following imperative:

*we call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms.*

We have seen increased awareness of the importance of active learning approaches among universities. From the MAA’s 2015 survey completed by chairs or undergraduate coordinators in departments of mathematics in the U.S. that offer a graduate degree in mathematics, 44% of respondents recognized active learning as “very important to having a successful precalculus/calculus sequence.” In contrast, only 15% consider themselves to be very successful at using active learning. This gap represents a significant need for information and assistance.

We were very fortunate to have six outstanding practitioners and advocates of active learning to speak to us on the first day of this conference. The first three, Ben Braun, Angie Hodge, and Mike Starbird spoke to the nature and importance of active learning and introduced Inquiry Based Learning, one of its purest and most effective forms. The second set of speakers, Charles Henderson, Tara Holm, and Dennis DeTurck addressed issues of implementation, how to overcome the inertia that keeps us in modes of instruction that we know only serve a small minority of our students. As the MAA survey illustrates, the issues are less about knowing what works than understanding how to tailor these approaches to a particular institutional situation and then to sustain them.

REFERENCES


David Bressoud is DeWitt Wallace Professor of Mathematics at Macalester College, a former President of the Mathematical Association of America, and a Fellow of the American Mathematical Society. He has published over 60 research articles in number theory, combinatorics, special functions, and mathematics education and authored or co-authored seven textbooks ranging from number theory to real analysis. He has served as PI for two national studies of Calculus: Characteristics of Successful Programs in College Calculus (NSF #0910240) and Progress through Calculus (NSF #1430540).

Assessment can take on many different forms and be part of both large- and small-scale projects. The panelists presented on how assessment can be used to measure active learning in general as well as for specific programs and classrooms. The first set of presentations focused on assessment and evaluation of mathematics instruction. Ted Mahavier spoke about assessment for the field in general and identified collaborations for future research studies that are necessary to support active learning in undergraduate mathematics courses. Nancy Ritze discussed the efforts that her community college is undertaking to measure student success and promote better student learning outcomes.

The second set of speakers, Mikhail Bouniav and Jerzy Mogilski presented on the use of formative assessments in active learning classrooms at their university. This session focused the use of assessment within the classroom to assess the learning gains of individual students.

The final set of speakers, Zachary Kornhauser and Susannah Klah, presented on how a center for teaching and learning on a university campus can promote active learning and assessment across campus. The focus of this hands-on session was on assessment for both within a specific classroom and also more generally, program- or university-wide.

The main take-away from the assessment panelists is that active learning is an important methodology for engaging with and teaching students. However, it is essential to measure the efficacy of the methodology through assessment at all levels: generally, program- or university-wide, and for individual students within an active learning classroom.

Doris Zahner is the Director of Test Development and Measurement Science at CAE where she oversees assessment development and psychometrics. Her research pertains to the international comparability of assessments in higher education and the validity of assessments in the college-to-career space. She holds a PhD in cognitive psychology and an MS in applied statistics from Teachers College, Columbia University. In addition to her responsibilities at CAE, Doris is an adjunct associate professor at Barnard College, Columbia University where she teaches statistics to undergraduate students in the social sciences and researches the use of diagrams in probability and mathematics problem solving.
Various definitions of active learning exist. For example, the 2016 Statement on Active Learning signed by presidents of member societies of the Conference Board for the Mathematical Sciences states that active learning (AL) refers to classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking [1]. However, there is not a unique definition of AL, either in popular use or in the research literature, and all existing definitions are inherently vague. No simple definition of AL can simultaneously and effectively address the range of AL techniques used across diverse classroom environments, institutional expectations for faculty in diverse employment contexts, and course and student learning outcomes across different institutions and departments. As a consequence, faculty, administrators, public-policy makers, student advocates, and other stakeholders in postsecondary mathematics (and STEM) education frequently “talk past” each other when discussing AL. I believe that better conversations occur when we define active learning by what it does in more specific contexts, specifically in the context of clear definitions of mathematical proficiency which inform student learning outcomes, thus informing our use of AL.

There are many existing frameworks for mathematical proficiency, e.g. the 2001 NRC report *Adding It Up* [2], which features a 5-strand model of proficiency, and the cognitive and content goals outlined in the 2015 MAA CUPM Curriculum Guide [3]. Once one of these frameworks for mathematical proficiency is selected, then student learning outcomes (SLOs) for a given course can be carefully developed. A robust set of SLOs will include intellectual, behavioral, and emotional aspects of student learning. With a clear vision of mathematical proficiency underlying articulate SLOs, we can then define an active learning method to be a classroom teaching technique in which students complete a task or activity directly supporting development in 1.) one or more student learning outcomes, 2.) one or more domains of mathematical proficiency, and 3.) one or more of the intellectual, behavioral, and emotional psychological domains. Our goal for each course should be to incorporate multiple AL techniques that collectively support development across all of our SLOs, domains of mathematical proficiency, and psychological domains.

As a simple example of this in the context of Calculus I, to support intellectual domain development, procedural fluency, and the ability to use derivatives, faculty can do the following: When working a simple example, take one minute to have the students compute the derivative of a polynomial independently. As a more extensive example of this in a number theory course, to support behavioral and emotional domain development, conceptual understanding, productive disposition, and productive collaboration with others, faculty can do the following: ask students to use Euclid’s proof of the infinitude of primes to produce as many new prime numbers as possible starting with only the prime 3. Students have three minutes to compute independently, then three minutes spent comparing their results with one or two of their neighbors in class, discussing the reason for why their lists are the same or different.
studying the aggregate impact of an AL technique, how do we determine whether or not a specific teaching technique in a specific classroom environment supports a specific SLO, proficiency domain, or psychological domain? To my knowledge, at this time many faculty using many AL techniques make these choices based on experience, intuition, and educated guesses informed by research in math education and psychology.


Benjamin Braun is an Associate Professor in the Department of Mathematics at the University of Kentucky, where he holds the Wimberly and Betty Royster Research Professorship. His mathematical research is in geometric and algebraic combinatorics, and he is active in mentoring graduate and undergraduate research students. His scholarly interests in teaching and learning include active learning, using writing in mathematics courses, pre-service teacher education, pedagogical use of the history of mathematics, and connections between mathematics education and educational psychology. He serves as a member-at-large on the American Mathematical Society Committee on Education and as the Editor-in-Chief of the American Mathematical Society blog On Teaching and Learning Mathematics.

ANGIE HODGE, UNIVERSITY OF NEBRASKA-OMAHA
ACTIVE LEARNING IN CALCULUS

Who am I?
• Dr. Angie Hodge (amhodge@unomaha.edu)
  Associate professor of mathematics and Haddix Chair of Mathematics Education
• Special Projects Coordinator for the Academy of Inquiry-Based Learning (AIBL)
• User of active learning (and inquiry-based learning) since August 2007

Why active learning in calculus when it is often implemented in upper-level classes?
• Calculus is considered the “gateway” course for many STEM disciplines
• Calculus study – (Bressoud, Carlson, Mesa, Rasmussen, 2013; Bressoud, 2015)
• Many students are not successful in Calculus I; many also struggle in Calculus II
• Desire to increase the quantity and quality of future mathematics teachers

What does active learning calculus mean?
• Students are activity engaged in mathematics
• Few traditional lectures are given (instead tactile activities and active learning worksheets are used)
• Activities selected to guide students into developing their own understandings

What does active learning calculus look like at the University of Nebraska Omaha?
• The classroom – start with a room well-suited for group work – tables!
• Class begins with daily student presentations of homework problems.
• Little introduction – worksheets distributed regularly with students working together.
What is the structure for group work in my active learning calculus courses?
- I have my students self-select groups of 3-4 students at the beginning of the semester
- The students shop around for the first 1-2 weeks of the semester
- Students are allowed to switch as they see fit, but most students don’t switch
- Other instructors switch on some consistent basis (e.g., after each exam)

What is the active learning part of the assessment?
- Students are graded on participation in class (5% of course grade)
- Student presentations required, as well as daily attendance in class

What are some helpful hints for active learning in calculus?
- Working as a team helped us with creating materials for the course and implementing them
- It was hard work finding and creating good activities and assessments
- It was helpful bouncing ideas off each other; sharing efforts REALLY made a difference!!

Where can you go to learn more?
- Calculus study: [https://works.bepress.com/david_bressoud/77/](https://works.bepress.com/david_bressoud/77/)

**Dr. Angie Hodge** is an associate professor of mathematics and the Haddix Community Chair of Mathematics Education at the University of Nebraska Omaha. She completed her graduate studies in mathematics education from Purdue University in West Lafayette, Indiana. She has taught courses in axiomatic geometry, calculus, differential equations, history of mathematics, mathematics methods courses for secondary mathematics majors, graduate mathematics education courses, and mathematics content courses for elementary school teachers. She also leads several outreach projects for both K-12 teachers and K-12 students, including a four-week summer camp for middle school girls in science technology, engineering and mathematics. In all of these venues, Hodge employed an inquiry-based/active learning approach. She has won teaching awards by using this method of teaching and also conducts research on active learning in both the university setting and in outreach settings. She has national recognition in inquiry-based learning and is a Special Programs Coordinator for the Academy of inquiry-Based Learning.

**MICHAEL STARBIRD, UNIVERSITY OF TEXAS AT AUSTIN WHAT DOES I.B.L. DO FOR PEOPLE?**
One of the common consequences of an Inquiry Based Learning experience is to raise the standard for what our students mean by understanding. When students do not understand mathematics, they not only do not understand the topic, they also do not know what “understanding” would mean. Those of us who have had a lot of experience in mathematics have a better idea of what it means to not understand something. If we don’t know a certain branch of abstract algebra, we know what that means. But if a student’s entire
level of understanding mathematics or anything else is quite limited, that student cannot really appreciate the gap between his or her current state and better understanding. Understanding is a continuum, and all of us sit at some point on that continuum with respect to everything we know. Inquiry Based Learning can help students to move forward on that continuum of understanding with respect to mathematics and also can help them to appreciate the possibility of improving their understanding in everything.

Not everyone has a consciousness of specific moments in their own experience when they took steps toward higher levels of precision and depths of understanding. For me, the transformative experience was an IBL class in graduate school. When I went to college I was a math major, but I never really understood anything extremely well. The classes were regular lecture style classes where I learned some things, did some homework, took the tests, and that was that. When I took an IBL class in graduate school, I actually proved theorems on my own. Only after that experience did I realize that it was possible to understand a subject at an entirely different level from what I had ever experienced before. At the end of that year I could take a blank piece of paper and write down the statements and proofs of every theorem of the year, as well as understanding why many promising attempts at proofs would not work. That was a completely unexpected and previously unexperienced possibility for me. When I think about the question, "What Can IBL Do For People?", one fundamental effect is to give students an experience of deeper understanding of a topic than they have had before. That experience can transform their perspective about their own level and potential level of understanding anything.

In a lecture experience it’s very clear what the role of the student is compared to the role of the instructor. Students sit there and try to remember and understand what is being said, whereas in a more Active Learning style of instruction, students are constructing knowledge by actually figuring things out; they are explaining ideas to one another; they are making mistakes and learning from those missteps; they are developing a community; they are having emotional responses to success, failure, and personal growth.

The effect of a well-constructed Active Learning experience goes beyond the mathematical content of the course. The real goals of education involve what students keep for life—curiosity, self-confidence to tackle challenges in all parts of life, and embracing the idea that increasingly deeper understanding is a lifetime adventure. Inquiry Based Learning experiences often help students to adopt more effective habits of mind that improve their success in every aspect of academic and non-academic life.


Michael Starbird is a University Distinguished Teaching Professor of Mathematics at The University of Texas at Austin. He has been at UT his whole career except for leaves, including to the Institute for Advanced Study in Princeton, New Jersey and the Jet Propulsion Laboratory in Pasadena, California. He has received more than fifteen teaching awards including the Mathematical Association of America’s 2007 national teaching award, the Minnie Stevens Piper Professor statewide award, the UT Regents’ Outstanding Teaching Award, and most of the UT-wide teaching awards. He has given
hundreds of lectures and dozens of workshops on effective teaching and effective thinking. He has produced DVD courses for The Teaching Company in the Great Courses Series on calculus, statistics, probability, geometry, and the joy of thinking. He co-authored, with Edward Burger, the textbook *The Heart of Mathematics: An Invitation to Effective Thinking* and has co-authored two Inquiry Based Learning textbooks. His recent book with co-author Edward Burger is *The 5 Elements of Effective Thinking*.

Dennis DeTurck, University of Pennsylvania

**Supporting Sustainable Active Learning—Challenges & Opportunities**

Establishing significant, permanent and systemic change in something as fundamental as how we teach undergraduates requires considerable sustained and coordinated effort from many levels. This report gives a dean’s perspective on the development, implementation and acceptance of active-learning strategies in STEM classes at the University of Pennsylvania.

Some of the essential ingredients are obvious: a nucleus of faculty members from several disciplines who are dissatisfied with the status quo and who wish to experiment with methods that have been shown to be effective; support of the administration for experimentation with and facilitation of new approaches; a forum for communication among early adopters and a way to attract other faculty members to use the new methods; and construction of appropriate classroom spaces.

All of these are necessary but often not sufficient for widespread change. The conservative (in the literal sense) tendencies of faculty members about pedagogy, as well as their healthy skepticism about what might seem to them to be the latest educational fad, play out in their claims that “our institution is not like the ones where these methods were validated”, or “we don’t have those problems here”, or even an outright “Not Invented Here” reaction.

At Penn, additional validation came via a grant from the Association of American Universities, and the simple act of “branding” what we were doing as SAIL (for Structured Active In-Class Learning) has helped overcome some of this resistance. Coordinated communication from the offices of the Vice Provost for Education, the dean of the College of Arts and Sciences, the dean of Engineering, and our Center for Teaching and Learning provided essential support.

But for many faculty members, the most compelling reason to become part of the SAIL effort was a report from the university’s own Faculty Council on Access and Achievement (FCAA), which showed that at Penn there are significant differential outcomes in graduation rates, persistence in science, and even grades in gateway courses between majority and minority students; between men and women; and between students of high and lower socio-economic status. These discrepancies persist even after adjusting for indicators of level of preparation (especially in mathematics) as indicated by standardized testing data and our own diagnostics.

Starting in 2012 by reaching about 100 students in SAIL classes, we’ve expanded to just over 2000 (spring enrollments estimated) in the 2016-17 academic year. Some of the increase is attributable to the addition of suitable teaching spaces, as well as the results of an assessment program carried out by CTL and our Graduate
School of Education, that shows that participation in SAIL courses have had a significant positive impact on students’ persistence in STEM.

Dennis DeTurck is the Dean of the College of Arts and Sciences and Professor of Mathematics at the University of Pennsylvania. His mathematical interests focus on differential geometry and partial differential equations. He is the managing editor of the AMS Contemporary Mathematics book series, and with coauthors including a graduate student and an undergraduate won MAA’s Chauvenet prize in 2012. He was the founding director of Penn’s Moelis Access Science outreach program, and is the faculty director of one of the college houses on campus.

CHARLES HENDERSON, WESTERN MICHIGAN UNIVERSITY  
OBSTACLES TO IMPLEMENTATION OF ACTIVE LEARNING IN MATHEMATICS: ARE WE USING THE RIGHT CHANGE STRATEGIES?

There is a knowledge-practice gap in undergraduate STEM education. We have known for a long time that traditional undergraduate STEM instruction results in poor student outcomes. Substantial empirical research has shown that a wide variety of student outcomes can be improved when instructors move from traditional, transmission-style instruction to more student-centered, interactive instruction. However, although considerable time and money has gone into developing and disseminating research-based pedagogy and curricula, available evidence suggests that these reform efforts are having only a marginal impact.

Change agents in higher education typically attempt to bridge this gap by developing stronger evidence of the efficacy of active learning and telling more instructors about this evidence. This type of change strategy, focused on convincing individual instructors through rational arguments, is not sufficient to bring about large-scale change. Focusing only on individuals does not change the barriers to active learning that are embedded in the cultures and structures within which these individuals work. This often leads to inappropriate use and discontinuation. For example, in a survey study of 722 US undergraduate physics instructors, approximately 1/3 of instructors who try a research-based instructional strategy self-report that they discontinue use of that strategy. And, the majority of the self-reported users do not use the instructional strategy as recommended by the developer.

Two types of change strategies focused on environments and structures, rather than solely on individuals, can be more successful in promoting sustainable change: developing policy, and developing shared vision. Kotter’s eight stage leadership model is an example of a prescribed approach to change that falls within the developing policy category. The change agent is a formal leader and begins the process by developing a vision and then motivating others to follow this vision. During the change process, the leader provides resources and rewards to individuals within the organization in order to support the desired changes. Complexity Leadership Theory is an example of an emergent approach to change that falls within the developing shared vision strategy category. It targets all levels of the organization to promote the development of new ideas. Change agents support the emergence of new ideas by disrupting existing patterns and encouraging novelty. Good ideas that emerge from the resulting interactions are communicated to formal leaders.

It is important to emphasize that there is no ‘best’ change strategy. Selecting an appropriate change strategy depends on the goals, resources, and history of each
situation. Much is known about effective practices for each type of change strategy and this knowledge is not widely applied in change initiatives.


**Charles Henderson** is a Professor at Western Michigan University, with a joint appointment between the Physics Department and the WMU Mallinson Institute for Science Education. He is the co-founder and co-director of the WMU Center for Research on Instructional Change in Postsecondary Education. His research program focuses on understanding and promoting instructional change in higher education, with an emphasis on improving undergraduate STEM instruction. Dr. Henderson’s work has been supported by over $7M in external grants and has resulted in many publications. In spring 2010, he was a Fulbright Scholar with the Finnish Institute for Educational Research at the University of Jyväskylä, Finland. Dr. Henderson is the senior editor for the journal *Physical Review Special Topics—Physics Education Research* and has served on two National Academy of Sciences Committees: Undergraduate Physics Education Research and Implementation, and Developing Indicators for Undergraduate STEM education.

**TARA HOLM, CORNELL UNIVERSITY TRANSFORMING POST-SECONDARY MATHEMATICS (TPSE MATH)**

The education landscape has changed dramatically in the last half century. Higher education has become essential to economic mobility. At the same time, colleges, universities, and students are under severe financial pressure. And new pedagogies and technologies allow us to reach students in many more ways. These and other forces will change higher education. Mathematicians must play a central part. If we opt out, we risk losing the substantial role that mathematics departments currently play, and we endanger the health of the
US mathematical sciences research enterprise. I introduced and described the work of mathematicians and mathematics educators in the group *Transforming Post-Secondary Education in Mathematics* (TPSE Math or TPSE, for short, pronounced “tipsy”). We aim to coordinate and drive constructive change in education in the mathematical sciences at colleges and universities across the nation. We are just beginning this work, but we aim to build on the successes of the national mathematical sciences community.

The last period of dramatic change in high school and college mathematics curricula began in the 1950s. In 1957, the Soviets launched Sputnik I, the first satellite to go into orbit. Four months later, the US launched its first successful satellite, Explorer I. The Cold War Space Race had begun. In that era of unprecedented public support for science education, calculus became the ultimate goal of high school mathematics. Supported in part by the Ford Foundation, AP calculus came into being. Since then, the exam has shifted to being a test of calculus knowledge rather than more general problem solving. The variety of mathematics relevant to the world has expanded remarkably in the 60 years since then. We must open new pathways to offer students the mathematics they need. This is a particular challenge in mathematics, where theories do not become false or go out of fashion.

Generating systemic change is a notoriously complex challenge. Fortunately, there are models that have been successful in academia and can be adapted for the mathematical sciences community. In the Life Sciences and in Physics, curricula and pedagogy are now better adjusted to foster students’ conceptual understanding of the science. In both cases, real progress occurred only after the communities came together to articulate a coherent vision. Disconnected innovations are insufficient to transform the entire field.

In mathematics there are successful programs that we can build upon to facilitate successful propagation of change. Still, mathematics is different from the other sciences. In the physical sciences, where research is dependent on expensive equipment and experiments, the community decides with funding agencies on the top research priorities for the field. As a consequence, the scientists are more accustomed to work together as a community. A key TPSE goal is to enhance the existing structures within the mathematical sciences community to ensure this necessary community-wide progress. We urge you to participate in upcoming TPSE meetings and to join our efforts!

*Tara Holm* is Professor of Mathematics at Cornell University. She is a member of the AMS Committee on Education, chairing it from 2012 to 2016, and a member of the Board of Governors of TPSE Math. She has also served on the leadership team of the MAA Common Vision project, on the Executive Committee of the Association for Women in Mathematics, and on the Executive Council of the AMS. Holm is the President/CEO of Pro Mathematica Arte, the corporation that runs mathematics study abroad programs in Budapest, Hungary, for North American students. She conducts research in symplectic geometry, algebraic geometry and topology, and she has mentored two graduate students to complete PhDs, with three more in the pipeline. Holm also mentors undergraduate and high school students. She is a Fellow of the AMS and a Project NExT Fellow (Sepia Dot) of the MAA.
The teaching and learning of undergraduate mathematics has received increased attention in recent years. Multiple growing communities of professionals have re-envisioned how core proof courses such as real analysis might best be taught. The desire to give students responsibility for discovering key course content concepts and the opportunity to engage in authentic mathematical research at their level, generally referred to as inquiry-based learning (IBL), unifies many of these efforts. This workshop convened experts in education research, curriculum development, instruction, faculty development, and assessment in IBL undergraduate real analysis. We surveyed the state of multiple perspectives on the field and existing connections across these areas of expertise. We identified and framed future collaborations to refine research-based studies and develop a research agenda responsive to existing needs regarding IBL practice. The primary focus of the workshop was to refine pertinent, tractable research questions and design consequent high-quality studies to address these questions. We identified four areas in which additional research efforts are needed to support IBL instruction in undergraduate real analysis. The workshop resulted in collaborative teams with diverse expertise to pursue necessary resources and tackle the critical research questions identified in the workshop.

What follows are the areas for which we developed research questions.

- **Instructor choices**
  - Establishing a productive classroom culture
  - Learning through proof presentations
- **Persistence & identity**
  - Exploratory study of student development
  - Case study of the impact of IBL on student development
  - Benefits of Modified Moore Method (MMM) over Lecture for Strong Students
- **Problem sequences & learning trajectories**
  - Intellectual cross-training
  - Strategic Walls
  - Beyond proof
  - Designing in the Zone of Proximal Development

A Professor of Mathematics at Lamar University, Ted Mahavier's publications span mathematical research, mathematics education, inquiry-based course notes and two books. He has served as PI or co-PI on seventeen funded grant proposals totaling more than two million dollars. As co-founder and Managing Editor for The Journal of Inquiry-Based Learning in Mathematics, he manages the only journal dedicated to publishing refereed inquiry-based course notes in mathematics. He is editor and co-author of *The Moore Method: A Pathway to Learner-Centered Instruction*, the definitive “how-to” manual for inquiry-based learning in mathematics. He is a nationally recognized speaker on IBL and four faculty he has mentored have earned the MAA Sectional teaching award.
NANCY RITZE, BRONX COMMUNITY COLLEGE

PROMOTING ACTIVE TEACHING AND LEARNING

Bronx Community College (BCC) of the City University of NY (CUNY) has developed a structured and comprehensive faculty development program that builds upon successful efforts recently undertaken by the college. This effort has been developed as a systemic approach to address the poor academic performance and persistence of at-risk BCC students. These faculty development activities are integrated into the College’s strategic plan and support the following goals: (1) Empower students to Succeed; and (2) Deepen Student Learning.

Recent findings in a major institutional self-study suggested that BCC students enter college without the skills and knowledge to be successful college students and new faculty members are equally ill-prepared to teach students who are not prepared for college. About one-half of the student population (53%) are first generation college students and 53% have an annual household income less than $20,000. Almost all (90%) of entering BCC freshmen require remediation in one or more basic skill area, with almost one-quarter (24%) needing remediation in all 3 skill areas.

This session provided concrete examples of BCC’s efforts to facilitate active teaching and learning to promote student success and deepen student learning, including the following examples.

Currently in its second year, the New Faculty Seminar was redesigned to include 3 major topics that are covered over the course of the entire first year of instruction at BCC: pedagogy, assessment and professional development at BCC. All faculty receive reassigned time to participate in the Seminar, which meets monthly and includes an intensive winter session. The section on pedagogy focuses on active learning and high impact practices that are successful with the student population at BCC. The faculty mentors model good pedagogy such as peer-to-peer work and flipped classrooms. All participating faculty create a teaching portfolio and an assessment project.

The College also has a First Year Seminar (FYS) for Students, which includes a number of high impact practices designed to improve student success in the first year and to function as an incubator for high impact teaching practices (active teaching and learning, use of e-portfolio, focus on learning for application). Once faculty across the curriculum participate in the training for and teaching of the First Year Seminar, they are more likely to use those pedagogies in other courses they teach. BCC engaged the Council for Aid to Education (CAE) in conducting Performance Task Workshops for selected populations of faculty, including a required session for those who are training to teach the First Year Seminar (FYS) next fall. These faculty learned about how to develop performance tasks for their courses which will help to better align teaching and learning with assessment. Another workshop was offered for faculty teaching special sections of courses and another is planned for faculty teaching developmental math courses.

Discussion Questions

- How to incentivize participation from all academic departments/disciplines?
- What strategies to specifically engage math faculty in active learning?
• Strategies to systematically support scholarship of teaching across the curricu-
lum?

Bibliography
• Alfieri, L., Brooks, P.J., Aldrich, J.J., & Tenenbaum, H/R (2011). Does discov-
ery-based instruction enhance learning? Journal of Educational Psychology, 103
(1), 1-18.
• Blumberg, P. 2009. Developing Learner-Centered Teaching: A Practical Guide
to Faculty. San Francisco: Jossey Bass.
• Karp, M.M., Bickerstaff, S., Rucks-Ahindiana, Z., Bork, R.H., Barragan, M., &
Edgecombe, N. (2012). College 101 courses for applied learning and student
and student success (CCRC Working Paper No. 49), New York, NY: Colum-
bia University, Teachers College, Community College Research Center.
• Karp, M.M., Raufman, J., Efthimiou, C., & Ritze,(2106) N. Revising a College
101 Course for Sustained Impact: Early Outcomes. Community College Jour-
nal of Research and Practice.
• Webber, Karen. “The Use of Learner Centered Assessment in US Colleges
and Universities.” Research in Higher Education 2012: 201-228

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Mikhail Bouniaev & Jerzy Mogilski, University of Texas at Rio Grande Valley Forma-
tive Assessment of Active Learning: Theo-
retical Perspective & Personal Experience

Based on the data analysis presented by Freeman and his co-
authors [2] who reviewed more than 300 published and un-
published studies on active learning, there is no doubt that active
learning can be very successful and students benefit from it. How-
ever, analyzing Freeman’s list of references (as well as results of
internet search), we found significantly fewer references to assess-
ment of learning compared to “organizing active learning” refer-
ces. So we can assume that either the science of assessment of
active learning in mathematics still has to be developed to reach the
same level as active learning per se, or there is no significant difference between
assessing traditional learning and active learning.

Most definitions of active learning from our point of view contain the following two
ideas: “information communicated to learner that is intended to modify his or her
thinking or behavior for the purpose of improving learning” [4]. This is a definition
from a “learner's perspective” complementing the definition from the instructor’s
perspective – “formative assessment is generally defined as assessment for the pur-
poses of instruction”[5]. In mathematics “active learning” and “inquiry based learn-
ing (IBL)” are closely related, though opinions vary regarding the extent to which
they are related or overlap.” [6,Part I]. We believe that this discussion with the goal
to establish some level of mutually acceptable perception of what active
learning is and how useful it is in light of the idea adopted by many scholars that there are four components of curriculum development: objectives (learning outcomes), content, strategies, and assessment [7], and all four components should be interconnected in the course of instructional design and implementation.

One of the challenges related to assessment of active learning is to find a helpful for practicing teachers/instructors answer to two questions about what and when to assess. We can’t complain that there is lack of the answers to both of these questions from theoretical perspectives. Though formative assessment can be organized in many different forms, some psychologists argue that formative assessment can employ three main methods for gathering data, namely, observation, test, and clinical interview [8]. By gathering data “…the teacher needs to learn about performance, thinking/knowledge, learning potential, and affect/motivation.” [9]. Bloom’s Taxonomy and its multidimensional modifications also provide some indications where to look for the answer to what to assess. More challenges occur in answering the second question “when or how often to collect data for formative assessment” There are some studies that show that efficacy of formative assessment depends on motive (students’ need), means (students are willing and able to use it), and opportunity (students receive it in time to use it) [10].

Ginsburg [9] argues that assessment should be “… based on psychological ideas and can be only as good as those ideas …. The theory should make sense to teacher … It need not deal with broad generalities, like constructivism. That … offers little insight into details of students behavior …”. We concur that organization of effective instruction including formative assessment should be based on solid learning theories, however, we also would like to “defend” social constructivism. Bouniaev and Connell in their paper on Social constructivism and Stage-by-Stage Development of Mental Actions Theory (SSDMA) recommend very similar specific (not general) strategies for organizing active learning of mathematics at various levels [11]. Active learning instructional strategies, including assessment strategies, that answer fundamental for the formative assessment questions “why, what and when” to assess, could be expressed within the SSDMA. The basics of SSDMA relevant to teaching collegiate mathematics will be briefly discussed following Bouniaev’s paper [12]. There is an apparent connection between the assessment of mental actions to be developed and the concepts of high/low level of thinking and cognitive demand [6,Part II].

In the last ten years we have been able to integrate the above mentioned ideas of formative assessment into various projects. Here we discuss briefly some of them: integration of formative assessment to teaching calculus, linear algebra and foundations of mathematics (graduate course); development of major specific labs for teaching calculus with labs designed for particular science/engineering majors with tracks in biology, chemistry, physics, engineering and computer science; formative assessment in the context of teaching new graduate classes and workshops. We would like to dwell more on the development of formative assessment methods within the new graduate courses for mathematics education majors “Collegiate Teaching of Mathematics”, and “College readiness and curriculum alignment”. Active learning and formative assessment strategies were integrated in the summer institute for in service teacher with main theme of study “College Readiness and Curriculum alignment”. Two of these projects, “Major Specific Lab Component of Calculus” and “College Readiness and Curriculum alignment” have been developed in the framework of the Department of Education MSEIP grant “Readiness, Recruitment, Retention, Graduation – Four Dimensions for Achieving Hispanic Student STEM Success.

In the published literature we came across a statement “active learning requires active assessment” [3]. We would like our audience to discuss this statement and share substantiated thoughts on it. The second question we would like to be discussed is the following. In planning assessment we should keep in mind objectives, strategies, and content.
What affects assessment the most?

Quotes from the summary of the discussions within 11 groups in the audience.

- “…We talked about the difference between what we viewed as active assessment and assessment. But we weren't really sure what was meant by the word active assessment and we had some discussion about that.”; – “… we decided yes, we agree with the statement but active learning doesn't only use active assessment. We all agreed that objectives affect our formal assessment strategies the most…”; – “…At our table we said that active assessment can be active learning with a grade attached but maybe it shouldn't have a grade…”; – “…We were trying to answer the question in the sense of active learning as a collection of techniques to facilitate the creation of shared meaning between the students and the teachers and active assessment as one of these tools.”; - “… As we were struggling with what active assessment was, we thought of an assessment opportunity that provided a back and forth feedback. ..”; ---“…formative assessment is important for the instructor, the student, and the other students to see what a student is thinking about…”; - “… We had a robust discussion about what the question was. The first thing you have to ask is what I am assessing. Am I assessing student learning, am I assessing teacher implementation of active learning tasks, am I assessing the efficacy of a given task across multiple teachers?”; - “… We had a discussion of examples. One of my colleagues has basically perfected a lovely thing that is an active assessment that isn't totally separate from active learning. We use it even in non-active learning classes…”.

References
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SUZANNA KLAFF, COLUMBIA UNIVERSITY

ACTIVE LEARNING AND ASSESSMENT

How do Centers for Teaching and Learning (CTL) promote active learning and assessment?

CTLs support instructors as they integrate active learning strategies and assessment into their classrooms. Doing so involves modeling practices, raising instructor awareness to strategies informed by the literature, and working with instructors as they design learner-centered experiences. Whether through institutes, workshops, or consultations, our approach is to encourage instructors to “begin with the end in mind” that is to design their courses or units of study using backward design which asks them to (1) identify desired student learning outcomes, (2) determine evidence of student learning, and (3) plan learning experiences and instructions.

Instructors are encouraged to align learning objectives with assessments, and take a holistic view of active learning. For deep meaningful learning to be achieved, students engage with information and ideas, apply their knowledge and skills through in-class and out-of-class experiences, and reflect on what and how they have learned. This framework is intended to inform instructor course planning and selection of active learning strategies.

Assessment in the active classroom involves summative assessment (high stakes evaluation of student learning), as well as formative assessment (low stakes monitoring of student learning), such as the use of CAT’s (classroom assessment techniques). Collecting this information allows instructors to analyze and make adjustments to instruction, thus closing the assessment loop.

It is good practice to collect student feedback on what worked well and what could be improved to enhance learning. This along with ongoing self-reflection and peer review of teaching can provide a more robust evaluation of teaching effectiveness.

How can instructors take a scholarly approach to their teaching, and advance the field?

A scholarly approach to teaching involves seeking out the literature and engaging in teaching-as-research a “deliberate, systematic, and reflective use of research
methods by instructors to develop and implement teaching practices that advance the learning experiences and outcomes of both students and teachers.” Inquiry into the classroom contributes to the scholarship of teaching and learning (SoTL) and advances effective mathematics teaching and learning. Practitioners interested in getting started with SoTL should seek out resources such as how-to guide and the MAA Press’ “Doing the Scholarship of Teaching and Learning in Mathematics” (2015) edited by Deward and Bennett.


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ZACHARY KORNHAUSER, COLUMBIA UNIVERSITY

ACTIVE LEARNING AND ASSESSMENT

Issues in college STEM education

STEM education in the United States has come under intense scrutiny in recent years due to the documentation of many troubling indicators. As detailed in a report submitted to the president, fewer than 40% of students intending to major in a STEM field graduate with a STEM degree, and attrition out of STEM is greater for women and minorities who are already underrepresented in STEM classrooms (PCAST, 2012). Evidence suggests that issues of teaching and learning contribute to the large rates of attrition from STEM majors, as high performing students often find STEM courses uninspiring, and low performing students perceive them as unwelcoming. Increases in active learning strategies has been proffered as a suggestion to address some of the issues facing
STEM education at the classroom level.

Active learning
Active learning does not have a single definition, but it can be understood as practices that require students to be participating agents in the learning process, and not receptacles who record or absorb information transmitted to them by the instructor (Bonwell & Eison, 1990). In active classrooms, emphasis is placed on engaging students and developing their high order cognitive faculties. A large body of research indicates that classrooms which promote active learning increase student performance along many indicators (Freeman et al., 2014).

Assessing presence and impact of active learning
Multiple methods exist for assessing the presence of active learning in courses, or the impact that active learning methods have on students. Survey instruments, such as the CLASSE (Ouimet & Smallwood, 2005), have been used to examine indicators of active learning in classrooms. The CLASSE asks students to indicate the frequency with which they engaged in certain educational practices, and compares these responses to instructors’ perceptions regarding the importance of those practices. This comparison may yield information about practices that may not be occurring as frequently as expected. Observational instruments are also commonly used to document evidence of active learning in classrooms. One commonly used instrument, the Classroom Observation Protocol for Undergraduate STEM (COPUS; Smith, Jones, Gilbert, & Wieman, 2013) categorizes classrooms by the behaviors that students and instructors exhibit. As active learning classrooms tend to share certain common characteristics, this instrument can be used to assess the extent to which these characteristics are present. Tests, such as the Classroom Test of Scientific reasoning (Lawson, 1978), can be used to assess the impact of active classrooms on students’ skills. This instrument assesses scientific reasoning patterns, which may be promoted in active classrooms. Scales, such as the Science Motivation Questionnaire (SMQ; Glynn, 2011), can also be used to measure the impact of active classrooms. This tool assesses students’ motivation to learn science, which is a product of a number of factors including instructor behavior. (Black & Deci, 2000).

References

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